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常利率下有阈红利边界的 Erlang(2) 风险模型 的罚金折现期望函数*

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摘 要: 为了精确地描述风险投资商实际的经营状况, 本文将一般的 Erlang(2) 风险模型推广为常利率下有阈红利边界的 Erlang(2) 风险模型。首先利用全概率公式对风险过程进行分析, 得到了模型的罚金折现期望函数所满足的积分-微分方程及积分方程, 然后在不带利率时将积分方程简化为“第二类非其次 Volterra 积分方程”, 给出了罚金折现期望函数的确切表达式, 最后给出了不带利率时模型的破产概率及破产前瞬时盈余和破产赤字的联合分布的表达式。

关键词: Erlang(2) 风险过程; 罚金折现期望函数; 阈红利边界; 积分-微分方程

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1 引言

随着对风险理论的深入研究, 文献 [1] 首次定义了经典风险模型的罚金折现期望函数, 并由此得到了破产概率及破产前瞬时盈余和破产赤字的联合分布的确切表达式。近年来对风险模型罚金折现期望函数的研究逐渐成为风险理论研究的热点之一, 学者们对经典风险模型做了不同程度的推广: 一种推广是假定理赔时间间隔服从 Erlang(2) 分布 (如文献 [2-6]); 另一种推广是考虑了利率或红利对收益的影响 (如文献 [7-10])。文献 [11] 将两者结合考虑了常利率下的 Erlang(2) 风险模型; 文献 [12] 考虑了常利率下的 Erlang(2) 风险模型的罚金折现期望函数, 给出了其满足的积分-微分方程及其 Laplace 变换满足的二阶微分方程, 且在不带利率时给出了罚金折现期望函数的确切表达式。本文考虑常利率下有阈红利边界的 Erlang(2) 风险模型, 即常利率下理赔时间间隔服从 Erlang(2) 分布, 给定一常数 $b > 0$, 当盈余过程 $0 \leq U(t) < b$ 时不向股东发放红利, 当 $U(t) \geq b$ 时以速率 r 向股东发放红利。这种经营方式既可以吸引顾客又可以减小公司的破产概率, 因而更具有现实意义。

2 模型介绍

设 (Ω, \mathcal{F}, P) 为包含本文中涉及的所有随机变量的完备概率测度空间。通常的 Erlang(2) 风险模型为

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X(i), \quad t > 0, \quad (1)$$

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其中 $u \geq 0$ 为初始盈余, $c > 0$ 为保费率, $N(t)$ 为到 t 时刻为止理赔发生的次数, $\{T_i, i \geq 1\}$ 为两次相邻理赔之间的时间间隔, 即

$$N(t) = \max\{n : T_1 + T_2 + \cdots + T_n \leq t\},$$

T_i 服从 Erlang(2) 分布, 密度函数为 $k(t) = \beta^2 t e^{-\beta t}$, $t > 0$, $\beta > 0$. $\{X_i, i \geq 1\}$ 是分布函数为 $F(x)$ 的独立同分布非负随机变量序列, 文中假设 $\{T_i, i \geq 1\}$, $\{X_i, i \geq 1\}$, $\{N(t), t \geq 0\}$ 两两相互独立. 本文考虑的常利率下有阈红利边界的 Erlang(2) 风险模型定义如下

$$U_\delta(t) = \begin{cases} u e^{\delta t} + c_1 \int_0^t e^{\delta v} dv - \int_0^t e^{\delta(t-v)} dS(v), & 0 \leq U_\delta(t) < b, \\ u e^{\delta t} + c_2 \int_0^t e^{\delta v} dv - \int_0^t e^{\delta(t-v)} dS(v), & U_\delta(t) \geq b, \end{cases} \quad (2)$$

其中

$$S(t) = \sum_{i=1}^{N(t)} X_i, \quad t > 0,$$

$c_1 > 0$ 是单位时间内的保费收入, $c_2 = c_1 - r$, r 是红利率. 模型 (2) 的破产时刻定义为: $T_\delta = \inf\{t \geq 0, U_\delta(t) < 0\}$, 若对所有的 t , 有 $U_\delta(t) \geq 0$, 则 $T_\delta = \infty$. 模型 (2) 的破产概率定义为: $\psi_\delta(u) = P\{T_\delta < \infty | U_\delta(0) = u\}$, 模型 (2) 的罚金折现期望函数定义为

$$\phi_\delta(u) = E\{e^{-\alpha T_\delta} \omega(U_\delta(T_\delta^-), |U_\delta(T_\delta)|) I(T_\delta < \infty) | U_\delta(0) = u\}. \quad (3)$$

其中 $I(\cdot)$ 为示性函数, α 是非负常数, $e^{-\alpha T_\delta}$ 为折罚因子, $U_\delta(T_\delta^-)$ 为破产前瞬时盈余, $|U_\delta(T_\delta)|$ 为破产赤字, 对任意 $0 \leq x_1, x_2 \leq \infty$, $\omega(x_1, x_2)$ 是非负有界函数. 为方便叙述, 将 $\psi_\delta(u)$ 记为如下形式

$$\phi_\delta(u) = \begin{cases} \phi_{1,\delta}(u), & 0 \leq u < b, \\ \phi_{2,\delta}(u), & u \geq b. \end{cases} \quad (4)$$

显然, 当 $\alpha = 0$, $\omega(x_1, x_2) = 1$ 时, (3) 式即为破产概率 $\psi_\delta(u)$; 当 $\alpha = 0$, $\omega(x_1, x_2) = I(x_1 < x, x_2 < y)$ 时, (3) 式则变为破产前瞬时盈余和破产赤字的联合分布.

3 主要结果

本节给出了 (4) 式满足的积分-微分及积分方程, 且当 $\delta = 0$ 时, 给出了积分方程的显式解.

定理 1 风险模型 (2) 的罚金折现期望函数 $\phi_\delta(u)$ 满足下面的积分-微分方程

$$\begin{aligned} (u\delta + c_1)^2 \phi_{1,\delta}''(u) &= (2\beta + 2\alpha - \delta)(u\delta + c_1) \phi_{1,\delta}'(u) - (\alpha + \beta)^2 \phi_{1,\delta}(u) \\ &\quad + \beta^2 \int_0^u \phi_{1,\delta}(u-x) dF(x) + \beta^2 \xi(u), \quad 0 \leq u < b; \end{aligned} \quad (5)$$

$$\begin{aligned} (u\delta + c_2)^2 \phi_{2,\delta}''(u) &= (2\beta + 2\alpha - \delta)(u\delta + c_2) \phi_{2,\delta}'(u) \\ &\quad - (\alpha + \beta)^2 \phi_{2,\delta}(u) + \beta^2 \int_{u-b}^u \phi_{1,\delta}(u-x) dF(x) \\ &\quad + \beta^2 \int_0^{u-b} \phi_{2,\delta}(u-x) dF(x) + \beta^2 \xi(u), \quad 0 \leq u \leq b. \end{aligned} \quad (6)$$

其中

$$\xi(t) = \int_t^\infty \omega(t, x-t) dF(x).$$

证明 考虑首次理赔发生的时刻 $T_1 = t$ 和首次理赔额 $X_1 = x$, 有两种可能的情况:

(i) 当

$$0 \leq t < \frac{1}{\delta} \ln [(b\delta + c_1)/(u\delta + c_1)]$$

时, t 时刻的瞬时盈余额 $0 \leq U_\delta(t^-) < b$, 且在 t 时刻有两种可能: 大于 0 小于 b 或小于 0;

(ii) 当

$$t \geq \frac{1}{\delta} \ln [(b\delta + c_1)/(u\delta + c_1)]$$

时, t 时刻的盈余额 $U_\delta(t^-) \geq b$, 且在 t 时刻盈余额有三种可能: 小于 0 或 0 和 b 之间或大于 b .

由上述讨论, 当 $0 \leq u < b$ 时, 由全概率公式得

$$\begin{aligned} \phi_{1,\delta}(u) &= \int_0^{\frac{1}{\delta} \ln N} \beta^2 t e^{-(\beta+\alpha)t} \gamma \left(u e^{\delta t} + c_1 \int_0^t e^{\delta v} dv \right) dt \\ &\quad + \int_{\frac{1}{\delta} \ln N}^\infty \int_{b+c_2(t-\frac{1}{\delta} \ln N)}^\infty \beta^2 t e^{-(\beta+\alpha)t} \omega \left(b + c_2 \left(t - \frac{1}{\delta} \ln N \right), x - b - c_2 \left(t - \frac{1}{\delta} \ln N \right) \right) dF(x) dt \\ &\quad + \int_{\frac{1}{\delta} \ln N}^\infty \int_{c_2(t-\frac{1}{\delta} \ln N)}^{b+c_2(t-\frac{1}{\delta} \ln N)} \beta^2 t e^{-(\beta+\alpha)t} \phi_{1,\delta} \left(b + c_2 \left(t - \frac{1}{\delta} \ln N \right) - x \right) dF(x) dt \\ &\quad + \int_{\frac{1}{\delta} \ln N}^\infty \int_0^{c_2(t-\frac{1}{\delta} \ln N)} \beta^2 t e^{-(\beta+\alpha)t} \phi_{2,\delta} \left(b + c_2 \left(t - \frac{1}{\delta} \ln N \right) - x \right) dF(x) dt, \end{aligned} \quad (7)$$

其中

$$N = (b\delta + c_1)/(u\delta + c_1), \quad \gamma(t) = \int_0^t \phi_{1,\delta}(t-x) dF(x) + \xi(t), \quad \xi(t) = \int_t^\infty \omega(t, x-t) dF(x).$$

令 $M = b + c_2(t - \frac{1}{\delta} \ln N)$, 然后对 (7) 式两边关于 u 求导数得

$$\begin{aligned} (u\delta + c_1) \phi'_{1,\delta}(u) &= (\beta + \alpha) \phi_{1,\delta}(u) - \int_0^{\frac{1}{\delta} \ln N} \beta^2 t e^{-(\beta+\alpha)t} \gamma \left(u e^{\delta t} + c_1 \int_0^t e^{\delta v} dv \right) dt \\ &\quad - \int_{\frac{1}{\delta} \ln N}^\infty \int_M^\infty \beta^2 t e^{-(\beta+\alpha)t} \omega(M, x-M) dF(x) dt \\ &\quad - \int_{\frac{1}{\delta} \ln N}^\infty \int_{M-b}^M \beta^2 t e^{-(\beta+\alpha)t} \phi_{1,\delta}(M-x) dF(x) dt \\ &\quad - \int_{\frac{1}{\delta} \ln N}^\infty \int_0^{M-b} \beta^2 t e^{-(\beta+\alpha)t} \phi_{2,\delta}(M-x) dF(x) dt. \end{aligned} \quad (8)$$

对(8)式两边关于 u 求导数可得(5)式成立。同理,当 $b \leq u$ 时,利用全概率公式得

$$\begin{aligned}\phi_{2,\delta}(u) &= \int_0^\infty \int_{\eta(t)}^\infty \beta^2 t e^{-(\beta+\alpha)t} \omega(\eta(t), x - \eta(t)) dF(x) dt \\ &\quad + \int_0^\infty \int_{\eta(t)-b}^{\eta(t)} \beta^2 t e^{-(\beta+\alpha)t} \phi_{1,\delta}(\eta(t) - x) dF(x) dt \\ &\quad + \int_0^\infty \int_0^{\eta(t)-b} \beta^2 t e^{-(\beta+\alpha)t} \phi_{2,\delta}(\eta(t) - x) dF(x) dt,\end{aligned}\quad (9)$$

其中

$$\eta(t) = ue^{\delta t} + c_2 \int_0^t e^{\delta v} dv.$$

对(9)式两边关于 u 两次求导可得(6)式成立。

定理2 对任意的 $u > 0$, 风险模型(2)的罚金折现期望函数 $\phi_\delta(u)$ 满足下面的积分方程

$$\phi_\delta(u) = \begin{cases} \phi_{1,\delta}(u) = G_1(u) + \int_0^u K_1(u, v) \phi_{1,\delta}(v) dv, & 0 \leq u < b, \quad (i) \\ \phi_{2,\delta}(u) = G_2(u) + \int_b^u K_2(u, v) \phi_{2,\delta}(v) dv, & u \geq b, \quad (ii) \end{cases} \quad (10)$$

其中

$$G_1(u) = \left[\beta^2 \int_0^u (u-v) \xi(v) dv + M(\delta)u + c_1^2 \phi_{1,\delta}(0) \right] / (u\delta + c_1)^2,$$

$$\begin{aligned}M(\delta) &= \frac{1}{b} \left[(b\delta + c_1)^2 \phi_{1,\delta}(b) - c_1^2 \phi_{1,\delta}(0) - \int_0^b (a_1 + a_2 b + a_3 v) \phi_{1,\delta}(v) \right. \\ &\quad \left. - \beta^2 \int_0^b \int_0^v F(v-y) \phi_{1,\delta}(y) dy dv - \beta^2 \int_0^b (b-y) \xi(y) dy \right],\end{aligned}$$

$$a = 2\beta + 2\alpha + \delta, \quad a_1 = (a + 2\delta)c_1, \quad a_2 = -(\alpha + \beta + \delta)^2, \quad a_3 = (\alpha + \beta + 2\delta)^2,$$

$$K_1(u, v) = \left[a_1 + a_2 v + a_3 u + \int_v^u F(z-v) dz \right] / (u\delta + c_1)^2;$$

$$G_2(u) = \left[\beta^2 \int_b^u \left[\int_0^b F(v-y) \phi_{1,\delta}(y) dy + (u-v) \xi(v) \right] dv + b_1 u + b_2 \right] / (u\delta + c_2)^2,$$

$$b_1 = N(\delta) - \beta^2 \int_0^b F(b-v) \phi_{1,\delta}(v) dv, \quad b_2 = (b\delta + c_2)^2 \phi_{2,\delta}(b) - b b_1,$$

$$\begin{aligned}N(\delta) &= ((b+1)\delta + c_2)^2 \phi_{2,\delta}(b+1) - (b\delta + c_2)^2 \phi_{2,\delta}(b) - \beta^2 \int_b^{b+1} \int_b^v \xi(y) dy dv \\ &\quad - \beta^2 \int_b^{b+1} \int_0^b F(v-y) \phi_{1,\delta}(y) dy dv + \beta^2 \int_0^b F(b-v) \phi_{1,\delta}(v) dv \\ &\quad - \int_b^{b+1} \left[(a + 2\delta)c_2 + a_2(b+1) + (2\delta^2 + a\delta - a_2)v + \beta^2 \int_v^{b+1} F(z-v) dz \right] \phi_{2,\delta}(v) dv,\end{aligned}$$

$$K_2(u, v) = \left[(a + 2\delta)c_2 + a_2 u + (2\delta^2 + a\delta - a_2)v + \beta^2 \int_v^u F(z-v) dz \right] / (u\delta + c_2)^2.$$

证明 对 (5), (6) 式两边积分便得结论。

定理 3 当 $\delta = 0$ 时, 风险模型 (2) 的罚金折现期望函数 $\phi(u)$ 为

$$\phi(u) = \begin{cases} \phi_1(u) = g_1(u) + \int_0^u L(u, s)g_1(s)ds, & 0 \leq u < b; \\ \phi_2(u) = g_2(u) + \int_b^u N(u, s)g_2(s)ds, & u \geq b. \end{cases} \quad (11)$$

其中

$$g_1(x) = [\beta^2 \int_0^x (x-v)\xi(v)dv + [c_1^2\phi_1'(0) - 2qc_1\phi_1(0)]x + c_1^2\phi_1(0)]/c_1^2,$$

$$\phi_1'(0) = [\beta^2[(2q - c_1r_1)\tilde{\xi}(r_2) - (2q - c_1r_2)\tilde{\xi}(r_1)]]/[c_1^3(r_1 - r_2)],$$

$$\phi_1(0) = [\beta^2[\tilde{\xi}(r_2) - \tilde{\xi}(r_1)]]/[c_1^2(r_1 - r_2)], \quad q = \alpha + \beta,$$

$$L(u, s) = \sum_{m=1}^{\infty} l_m(u, s), \quad u > s \geq 0, \quad l_m(u, s) = \int_s^u K_1(u, t)l_{m-1}(t, s)dt, \quad m = 2, 3, \dots,$$

$$l_1(u, s) = K_1(u, s), \quad K_1(u, s) = [2qc_1 - q^2s + q^2u + \int_s^u F(z-s)dz]/c_1^2;$$

$$g_2(x) = [\beta^2 \int_b^x \left[\int_0^b F(v-y)\phi_1(y)dy + (x-v)\xi(v) \right] dv + d_1x + d_2]/c_2^2,$$

$$d_1 = c_1c_2\phi_1'(b) - 2qc_2\phi_1(b) - \beta^2 \int_0^b F(b-v)\phi_1(v)dv, \quad d_2 = c_1c_2\phi_1(b) - bd_1,$$

$$N(u, s) = \sum_{m=1}^{\infty} h_m(u, s), \quad u > s \geq 0, \quad h_m(u, s) = \int_s^u K_2(u, t)h_{m-1}(t, s)dt, \quad m = 2, 3, \dots,$$

$$h_1(u, s) = K_2(u, s), \quad K_2(u, s) = [2qc_2 - q^2(u-s) + \beta^2 \int_s^u F(z-s)dz]/c_2^2.$$

证明 当 $\delta = 0$ 且 $b = \infty$ 时, 记方程 (10) 中 (i) 式的解为 $\phi_1^\infty(u)$; $\delta = 0$ 时, 记方程 (i) 的解为 $\phi_1(u)$ 。因为 $\phi_1^\infty(u)$ 为方程 (i) 的特解, 故有 $\phi_1^\infty(0) = \phi_1(0)$, $\phi_1^{\infty'}(0) = \phi_1'(0)$ 。因此利用文献 [12] 中的结论可得

$$\phi_1^\infty(0) = [\beta^2[\tilde{\xi}(r_2) - \tilde{\xi}(r_1)]]/[c_1^2(r_1 - r_2)], \quad (12)$$

其中

$$\tilde{\xi}(s) = \int_0^\infty e^{-sx}\xi(x)dx,$$

r_1, r_2 为方程 $(c_1s - \alpha - \beta)^2 = \beta^2\tilde{f}(s)$ 的两个非负根, 且有 $0 \leq r_1 < (\alpha + \beta)/c_1 < c_2$, 这里

$$\tilde{f}(s) = \int_0^\infty e^{-sx}dF(x).$$

再利用文献 [12] 中定理 4 的证明可得

$$2(\alpha + \beta)c_1\phi_1^\infty(0) - c_1^2\phi_1^{\infty'}(0) = c_1^2r_2\phi_1^\infty(0) + \beta^2\tilde{\xi}(r_2), \quad (13)$$

从而

$$\phi_1^{\infty'}(0) = [\beta^2[(2q - c_1r_1)\tilde{\xi}(r_2) - (2q - c_1r_2)\tilde{\xi}(r_1)]]/[c_1^3(r_1 - r_2)], \quad q = \alpha + \beta. \quad (14)$$

将(12), (14)式代入方程(i), 则方程(i)变为“第二类非齐次 Volterra 积分方程”, 可得其解为(11)式中的 $\phi_1(u)$ 。

下面给出当 $\delta = 0$ 时, 方程(10)中(ii)的解。因为 $\phi(u)$ 在 $u = b$ 处是连续的, 故有 $\phi_1(b) = \phi_2(b)$ 。类似定理1中的证明可证得: $c_1\phi'_1(b) = c_2\phi'_2(b)$, 其中 $\phi_1(b), \phi'_1(b)$ 均可由(11)式中第一式得到。从而(ii)式亦可化为“第二类非齐次 Volterra 积分方程”, 于是可得其解为(11)式中的 $\phi_2(u)$ 。综上, 当 $\delta = 0$ 时, (11)式成立。

推论 1 当 $\delta = 0$ 时, 模型(2)的破产概率为

$$\psi(u) = \begin{cases} \psi_1(u) = g_1^*(u) + \int_0^u L^*(u, s)g_1^*(s)ds, & 0 \leq u < b; \\ \psi_2(u) = g_2^*(u) + \int_b^u N^*(u, s)g_2^*(s)ds, & u \geq b. \end{cases} \quad (15)$$

其中

$$g_1^*(x) = \left[\beta^2 \left(\frac{1}{2}x^2 - \int_0^x (x-v)F(v)dv \right) + [c_1^2\psi'(0) - 2\beta c_1\psi(0)] + c_1^2\psi(0) \right] / c_1^2,$$

$$\begin{aligned} \psi'(0) &= [\beta^2[(2\beta - c_1r_1)\tilde{F}(r_1) - (2\beta - c_1r_2)\tilde{F}(r_2)]] / [c_1^3(r_1s - r_2)] \\ &\quad + [\beta^2(2\beta - c_1(r_1 + r_2))] / [c_1^3r_1r_2], \end{aligned}$$

$$\psi_1(0) = [\beta^2[\tilde{F}(r_2) - \tilde{F}(r_1)]] / [c_1^2(r_1 - r_2)] + \beta^2 / (c_1^2r_1r_2),$$

$$L^*(u, s) = \sum_{m=1}^{\infty} l_m^*(u, s), \quad u > s \geq 0, \quad l_m^*(u, s) = \int_s^u K_1^*(u, t)l_{m-1}^*(t, s)dt, \quad m = 2, 3, \dots,$$

$$l_1^*(u, s) = K_1^*(u, s), \quad K_1^*(u, s) = \left[2\beta c_1 - \beta^2s + \beta^2u + \int_s^u F(z-s)dz \right] / c_1^2,$$

$$g_2^* = \left[\beta^2 \int_0^x \left[\int_0^b F(v-y)\psi_1(y)dy - (x-v)F(v) \right] dv + d_1^* + d_2^* \right] / c_2^2,$$

$$d_1^* = c_1c_2\psi'_1(b) - 2\beta c_2\psi_1(b) - \beta^2 \int_0^b F(b-v)\psi_1(v)dv - b,$$

$$d_2^* = c_1c_2\psi_1(b) - bd_1^* + \frac{1}{2}(u^2 + b^2),$$

$$N^*(u, s) = \sum_{m=1}^{\infty} h_m^*(u, s), \quad u > s \geq 0, \quad h_m^*(u, s) = \int_s^u K_2^*(u, t)h_{m-1}^*(t, s)dt, \quad m = 2, 3, \dots,$$

$$h_1^*(u, s) = K_2^*(u, s), \quad K_2^*(u, s) = \left[2\beta c_2 - \beta^2(u-s) + \beta^2 \int_s^u F(z-s)dz \right] / c_2^2.$$

证明 只需在(11)式中令 $\alpha = 0$, $\omega(x, y) = 1$ 即可。

推论 2 当 $\delta = 0$ 时, 模型(2)的破产前瞬时盈余和破产赤字的联合分布为

$$G(x, y|u) = \begin{cases} G_1(x, y|u) = g_1(x, y|u) + \int_0^u L^{**}(u, s)g_1(x, y|s)ds, & 0 \leq u < b; \\ G_2(x, y|u) = g_2(x, y|u) + \int_b^u N^{**}(u, s)g_2(x, y|s)ds, & u \geq b. \end{cases} \quad (16)$$

其中

$$\begin{aligned}
 g_1(x, y|u) &= \left[\beta^2 \int_0^x (u-v)[F(v+y) - F(v)]dv \right. \\
 &\quad \left. + [c_1^2 G_1'(x, y|0) - 2\beta c_1 G_1(x, y|0)]u + c_1^2 G_1(x, y|0) \right] / c_1^2, \\
 G_1'(x, y|0) &= \beta^2 \left[(2\beta - c_1 r_1) \int_0^x e^{-r_2 v} (F(v+y) - F(v))dv \right. \\
 &\quad \left. - (2\beta - c_1 r_2) \int_0^x e^{-r_1 v} (F(v+y) - F(v))dv \right] / [c_1^3 (r_1 - r_2)], \\
 G_1(x, y|0) &= \left[\beta^2 \int_0^x (e^{-r_2 v} - e^{-r_1 v}) (F(v+y) - F(v))dv \right] / [c_1^2 (r_1 - r_2)], \\
 L^{**}(u, s) &= \sum_{m=1}^{\infty} l_m^{**}(u, s), \quad u > s \geq 0, \\
 l_m^{**}(u, s) &= \int_s^u K_1^{**}(u, t) l_{m-1}^{**}(t, s) dt, \quad m = 2, 3, \dots, \\
 l_1^{**}(u, s) &= K_1^{**}(u, s), \quad K_1^{**}(u, s) = \left[2\beta c_1 - \beta^2 s + \beta^2 u + \int_s^u F(z-s) dz \right] / c_1^2, \\
 g_2(x, y|u) &= \left[\beta^2 \int_0^x \int_0^b F(v-s) G_1(x, y|s) ds dv \right. \\
 &\quad \left. + \beta^2 \int_b^x (u-v)[F(v+y) - F(v)]dv + d_1^{**}u + d_2^{**} \right] / c_2^2, \\
 d_1^{**} &= c_1 c_2 G_1'(x, y|b) - 2\beta c_2 G_1(x, y|b) - \beta^2 \int_0^b F(b-v) G_1(x, y|v) dv, \\
 d_2^{**} &= c_1 c_2 G_1(x, y|b) - b d_1^{**}, \quad N^{**}(u, s) = \sum_{m=1}^{\infty} h_m^{**}(u, s), \quad u > s \geq 0, \\
 h_m^{**}(u, s) &= \int_s^u K_2^{**}(u, t) h_{m-1}^{**}(t, s) dt, \quad m = 2, 3, \dots, \quad h_1^{**}(u, s) = K_2^{**}(u, s), \\
 K_2^{**}(u, s) &= \left[2\beta c_2 - \beta^2 (u-s) + \beta^2 \int_s^u F(z-s) dz \right] / c_2^2.
 \end{aligned}$$

证明 只需在 (11) 式中令 $\alpha = 0$, $\omega(x, y) = I(x_1 < x, x_2 < y)$ 即可。

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The Expected Discounted Penalty Function for the Erlang(2) Risk Model with a Threshold Strategy Under Constant Interest

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Abstract: In this paper, in order to describe the actual business venture operating conditions more accurately, we extend the Erlang(2) risk model to the Erlang(2) risk model with a threshold strategy under constant interest. Firstly, the integro-differential equations and the integral equations satisfied by the expected discounted penalty function are derived by analyzing the risk process and utilizing the total probability formula. Furthermore, the integral equations are reduced to the second kind of non-homogeneous Volterra integral equations when the interest rate is zero, and the explicit solutions to the integral equations are obtained. Finally, the explicit formulae for the ruin probability and the joint distribution of the surplus immediately before ruin and the deficit at ruin are given in the interest-free case.

Keywords: Erlang(2) risk model; the expected discounted penalty function; threshold strategy; integro-differential equations

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